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Зміст

МАТЕМАТИКА

Semko N.N., Skaskiv L.V., Yarovaya O.A. Linear groups saturated by subgroups of finite central dimension **3**

Tushev A.V. On induced modules over locally Abelian-by-polycyclic groups of finite rank **8**

ІНФОРМАТИКА ТА КІБЕРНЕТИКА

Поліщук О.Д. Редукція складності моделей мережевих структур та систем **12**

МЕХАНІКА

Мейш В.Ф., Мейш Ю.А., Бєлов Є.Д. До побудови чисельного алгоритму розв'язку динамічних задач теорії дискретно підкріплених конічних оболонок в неортогональній системі координат **19**

Селіванов М.Ф. Крайова тріщина із зоною зчеплення в ортотропному тілі **25**

МАТЕРІАЛОЗНАВСТВО

Наконечна О.І., Білявина Н.М., Остапенко Р.В., Курилюк А.М., Макара В.А. Магнітні властивості карбіду NiC_x з дефектною структурою типу сфалериту **36**

Contents

MATHEMATICS

Semko N.N., Skaskiv L.V., Yarovaya O.A. Linear groups saturated by subgroups of finite central dimension **3**

Tushev A.V. On induced modules over locally Abelian-by-polycyclic groups of finite rank **8**

INFORMATICS AND CYBERNETICS

Polishchuk O.D. Reduction of the complexity of models of network structures and systems **12**

MECHANICS

Meish V.F., Meish Yu.A., Belov Ye.D. To the construction of a numerical algorithm for solving the dynamic problems of the theory of discretely reinforced conic shells in a nonorthogonal coordinate system **19**

Selivanov M.F. An edge crack with cohesive zone in the orthotropic body **25**

MATERIALS SCIENCE

Nakonechna O.I., Belyavina N.N., Ostapenko R.V., Kuryliuk A.M., Makara V.A. Magnetic features of NiC_x carbide with the sphalerite-type defect structure **36**

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НАУКИ ПРО ЗЕМЛЮ

Куриленко В.С. Соляной диапиризм Днепровско-Донецкой впадины с позиций флюидодинамических представлений **43**

ХІМІЯ

Сачук О.В. Нанодисперсні $ZnO—MoO_3$ каталізатори селективного окиснення етанолу, синтезовані нетрадиційними методами **48**

Сірик О.О., Самченко Ю.М., Полторацька Т. П., Крикля С.О., Трохимчук А.К. Адсорбція барвників різної природи на пористих сорбентах на основі полівінілформалю **54**

Shevchenko I.V. Influence of solar energy on self-organization of water molecules. Diurnal, annual, and 11-year variations **61**

БІОЛОГІЯ

Максименко Л.О., Горб Т.Ю. Властивості низькомолекулярних каротоворицинів, виділених з природних ізолятів *Pectobacterium carotovorum* subsp. *carotovorum* **67**

Оберніхіна Н.В., Ніколаєв Р.О., Качковський О.Д., Ткачук З.Ю. π-Електронна афінність азотистих основ нуклеїнових кислот **75**

БІОФІЗИКА

Zdorevskyi O.O., Piatnytskyi D.V., Volkov S.N. On a possibility of the blocking of DNA specific recognition sites by hydrogen peroxide molecules during ion beam therapy **82**

GEOSCIENCES

Kurilenko V.S. Salt diapirism of the Dnieper-Donets depression from the standpoint of fluid dynamics representations. **43**

CHEMISTRY

Sachuk O.V. Nanodispersed $Zn—MoO_3$ catalysts of selective ethanol oxidation synthesized by nontraditional methods **48**

Siryk O.O., Samchenko Yu.M., Poltoratska T.P., Kryklya S.O., Trofimchuk A.K. Adsorption of dyes of different types on polyvinylformal-based porous sorbents. **54**

Shevchenko I.V. Influence of solar energy on self-organization of water molecules. Diurnal, annual, and 11-year variations **61**

BIOLOGY

Maksymenko L.A., Gorb T.Yu. Properties of the low molecular weight carotovoricins from the natural isolates *Pectobacterium carotovorum* subsp. *carotovorum* **67**

Obernikhina N.V., Nikolaev R.O., Kachkovsky O.D., Tkachuk Z.Yu. π-electron affinity of the nitrogenous bases of nucleic acids. **75**

BIOPHYSICS

Zdorevskyi O.O., Piatnytskyi D.V., Volkov S.N. On a possibility of the blocking of DNA specific recognition sites by hydrogen peroxide molecules during ion beam therapy **82**

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Linear groups saturated by subgroups of finite central dimension

Presented by Academician of the NAS of Ukraine A.M. Samoilenko

Let F be a field, A be a vector space over F, and G be a subgroup of GL(F, A). We say that G has a dense family of subgroups having finite central dimension, if, for every pair of subgroups H, K of G such that $H \leqslant K$ and H is not maximal in K, there exists a subgroup L of finite central dimension such that $H \leqslant L \leqslant K$ (we can note that L can match with one of the subgroups H or K). We study locally solvable linear groups with a dense family of subgroups having finite central dimension.

Keywords: *linear group, infinite groups, infinite-dimensional linear group, dense family of subgroups, locally soluble groups, finite central dimension.*

We recall that a group G that is isomorphic to a group of automorphisms of a vector space A over a field F is called a linear group. We denote the group of all such automorphisms by $\mathrm{GL}(F, A)$. If $\dim_F(A)$, the dimension of A over F, is finite, n, then we say that G is a *finite-dimensional* linear group. It is well known that $\mathrm{GL}(F, A)$ can be identified with the group of $n \times n$ matrices with entries in F. From the outset, finite-dimensional linear groups have played an important role in group theory. This is partly due to the correspondence mentioned above, but also because of the rich interplay between geometric and algebraic ideas associated with such groups.

The study of the subgroups of $\mathrm{GL}(F, A)$ in the case where A is infinite-dimensional over F has been much more limited and normally requires some additional restrictions. There is quite a large array of papers that show the effectiveness of applying various natural limitations for the study of infinite-dimensional linear groups (see survey articles [1—4]). One area that proved to be quite effective was the study of linear groups that have a very big family of subgroups having finite central dimension.

Let G be a subgroup of $GL(F, A)$ and $Z = C_A(G)$. It is not hard to see that a subspace Z is G-invariant, and G acts trivially on Z. Therefore, we see that G actually acts on the quotient-space A/Z.

Let G be a subgroup of $GL(F, A)$. Then the *central dimension of* G is a dimension of the quotient-space $A/\zeta_G(A)$.

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In particular, if $\dim_F(A/\zeta_G(A))$ is finite, then we will say that *G has finite central dimension*. According to the definition, linear groups having finite central dimension are quite close to finite-dimensional linear groups. Therefore, it was obvious to study the infinite-dimensional linear groups saturated by subgroups of finite central dimension. Among the works on this subject, we mention [5, 6, 1, 7—10].

Among the restrictions that play a significant role in the study of both finite and infinite groups, we can highlight the restriction associated with the presence of one or another family of dense subgroups in a group. Let P be a some property. We say that a group G has a dense family of subgroups having property P, if, for every pair of subgroups H, K of G such that $H \leqslant K$ and H is not maximal in K, there exists a subgroup L, having property P, such that $H \leqslant L \leqslant K$ (we note that L can match with one of the subgroups H or K).

Groups with different natural dense families have been considered by many authors (see, for example, [11—15]).

In this paper, we will apply this restriction to the study of infinite-dimensional linear groups.

Let F be a field, A be a vector space over F, and G be subgroup of $GL(F, A)$. We say that *G has a dense family of subgroups, having finite central dimension*, if, for every pair of subgroups H, K of G such that $H \leqslant K$ and H is not maximal in K, there exists a subgroup L of finite central dimension such that $H \leqslant L \leqslant K$ (we note that L can match with one of the subgroups H or K).

Note that infinite-dimensional linear groups, whose proper subgroups have finite central dimension, have this property. Locally soluble groups, whose proper subgroups have finite central dimension, were studied in paper [5]. Therefore, in this paper, the study of linear groups with a dense family of subgroups, having finite central dimension, will be conducted under the additional condition of their local solvability.

Lemma 1. *Let F be a field, A be a vector space over F, and G be subgroup of GL(F, A).*

(*i*) *If H, K are two subgroups of G such that $H \leqslant K$ and a subgroup K has finite central dimension, then the subgroup H has finite central dimension.*

(*ii*) *If H, K are two subgroups having finite central dimension, then the both subgroups $\langle H, K\rangle$ and $H \cap K$ have finite central dimension.*

(*iii*) *If G has finite central dimension and* $\mathrm{char}(F) = p$ *is a prime, then G includes a normal elementary Abelian p-subgroup L such that G/L is isomorphic to some subgroup of GL(n, F), where* $n = \dim_F(A/C_A(G))$.

(*iv*) *If G has finite central dimension and* $\mathrm{char}(F) = 0$, *then G includes a normal Abelian torsion-free subgroup L such that G/L is isomorphic to some subgroup of GL(n, F), where* $n = \dim_F(A/C_A(G))$.

Corollary 1. *Let F be a field, A be a vector space over F, having infinite dimension, and G be a subgroup of GL(F, A). If H, K are two subgroups of G such that $H \leqslant K$, and a subgroup H has infinite central dimension, then the subgroup K has infinite central dimension.*

Corollary 2. *Let F be a field, A be a vector space over F, having infinite dimension, and G be a subgroup of GL(F, A). If G has a dense family of subgroups having finite central dimension, then every subgroup of G having infinite central dimension is maximal.*

Lemma 2. *Let F be a field, A be a vector space over F, having infinite dimension, and G be a subgroup of GL(F, A). Suppose that G includes the subgroups B, K satisfying the following conditions:*

(*i*) *B is an infinite elementary Abelian p-subgroup, and K is a quasicyclic p-subgroup for some prime p;*

(*ii*) *B is K-invariant;*

(*iii*) *$B \cap K = \langle 1 \rangle$.*

If G has a dense family of subgroups having finite central dimension, then K has finite central dimension.

Corollary 3. *Let F be a field, A be a vector space over F, having infinite dimension, and G be a subgroup of $GL(F, A)$. Suppose that G includes the subgroups B, K satisfying the following conditions:*

(*i*) *B is an infinite elementary Abelian p-subgroup, and K is a quasicyclic p-subgroup for some prime p;*

(*ii*) *B is K-invariant;*

(*iii*) *$B \cap K$ is finite.*

If G has a dense family of subgroups having finite central dimension, then K has finite central dimension.

Lemma 3. *Let F be a field, A be a vector space over F, having infinite dimension, and G be a subgroup of $GL(F, A)$. Suppose that G includes the subgroups B, K satisfying the following conditions:*

(*i*) *B is an infinite elementary Abelian p-subgroup, and K is a quasicyclic q-subgroup, where p, q are primes and $p \neq q$;*

(*ii*) *B is K-invariant;*

If G has a dense family of subgroups having finite central dimension, then either K has finite central dimension or B is a minimal K-invariant subgroup.

Lemma 4. *Let F be a field, A be a vector space over F, having infinite dimension, and G be a locally soluble subgroup of $GL(F, A)$. Suppose that G includes a subgroup K, having infinite central dimension. If G has a dense family of subgroups having finite central dimension, then K satisfies the following conditions:*

(*i*) *K is a quasicyclic or cyclic p-subgroup for some prime p;*

(*ii*) *every proper subgroup of K has finite central dimension;*

(*iii*) *K is a maximal subgroup of G.*

Theorem. *Let F be a field, A be a vector space over F, having infinite dimension, and G be a locally soluble subgroup of $GL(F, A)$. Suppose that G has infinite central dimension. If G has a dense family of subgroups having finite central dimension, then G is a group of one of following types:*

(*i*) *G is a cyclic or quasicyclic p-group for some prime p;*

(*ii*) *$G = K \times L$, where K is a cyclic or quasicyclic p-group for some prime p, and L is a group of prime order;*

(*iii*) *$G = \langle a, b \mid |a| = 2^n, |b| = 2, a^b = a^t$, where $t = 1 + 2^{n-1}$, $n \geqslant 3 \rangle$;*

(*iv*) *$G = \langle a, b \mid |a| = 2^n, |b| = 2, a^b = a^t$, where $t = -1 + 2^{n-1}$, $n \geqslant 3 \rangle$;*

(*v*) *$G = \langle a, b \mid |a| = 2^n, |b| = 2, a^b = a^{-1} \rangle$;*

(*vi*) *$G = \langle a, b \mid |a| = 2^n, b^2 = a^t$, where $t = 2^{n-1}$, $a^b = a^{-1} \rangle$;*

(*vii*) *$G = \langle a, b \mid |a| = p^n, |b| = p, a^b = a^t$, where $t = 1 + p^{n-1}$, $n \geqslant 2$, p is an odd prime$\rangle$;*

(*viii*) *$G = \langle a \rangle \leftthreetimes \langle b \rangle$, $|a| = p^n$ where p is an odd prime, $|b| = q$, q is a prime, $q \neq p$;*

(*ix*) *$G = B \leftthreetimes \langle a \rangle$, $|a| = p^n$, $B = C_G(B)$ is an elementary Abelian q-subgroup, p and q are primes, $p \neq q$, B is a minimal normal subgroup of G;*

(*x*) $G = K \leftthreetimes \langle b \rangle$, *where* K *is a quasicyclic 2-subgroup,* $|b| = 2$ *and* $x^b = x^{-1}$ *for each element* $x \in K$;

(*xi*) $G = K\langle b \rangle$, *where* $K = \langle a_n \mid a_1{}^p = 1, a_{n+1}{}^p = a_n, n \in N \rangle$ *is a quasicyclic 2-subgroup,* $b^2 = a_1$ *and* $a_n{}^b = a_n{}^{-1}$, $n \geqslant 2$.

(*xii*) $G = K \leftthreetimes \langle b \rangle$, *where* K *is a quasicyclic* p*-subgroup,* p *is an odd prime,* $K = C_G(K)$, $|b| = q$ *is a prime such that* $p \neq q$;

(*xiii*) $G = Q \leftthreetimes K$, *where* K *is a quasicyclic* p*-subgroup,* $Q = C_G(Q)$ *is an elementary Abelian* q*-subgroup,* p, q *are primes,* $p \neq q$, Q *is a minimal normal subgroup of* G.

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ЛІНІЙНІ ГРУПИ З НАСИЧЕНИМИ ПІДГРУПАМИ
СКІНЧЕННОЇ ЦЕНТРАЛЬНОЇ РОЗМІРНОСТІ

Нехай F – поле, A – векторний простір над F, G — підгрупа $GL(F, A)$. Будемо говорити, що G має щільне сімейство підгруп, які мають скінченну центральну розмірність, якщо для кожної пари підгруп H, K з G

такої, що $H \leqslant K$ і H немаксимальна в K, існує така підгрупа L скінченної центральної розмірності, що $H \leqslant L \leqslant K$ (зазначимо, що L може збігатися з однією з підгруп H або K). У роботі описані локально розв'язні лінійні групи з щільним сімейством підгруп, що мають скінченну центральну розмірність.

***Ключові слова**: лінійна група, нескінченна група, нескінченно розмірна лінійна група, щільне сімейство підгруп, локально розв'язна група, скінченна центральна розмірність.*

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ЛИНЕЙНЫЕ ГРУППЫ С НАСЫЩЕННЫМИ ПОДГРУППАМИ КОНЕЧНОЙ ЦЕНТРАЛЬНОЙ РАЗМЕРНОСТИ

Пусть F — поле, A — векторное пространство над F, G — подгруппа $GL(F, A)$. Будем говорить, что G имеет плотное семейство подгрупп, имеющих конечную центральную размерность, если для каждой пары подгрупп H, K из G такой, что $H \leqslant K$ и H немаксимальная в K, существует такая подгруппа L конечной центральной размерности, что $H \leqslant L \leqslant K$ (отметим, что L может совпадать с одной из подгрупп H или K). В работе описаны локально разрешимые линейные группы с плотным семейством подгрупп, имеющих конечную центральную размерность.

***Ключевые слова**: линейная группа, бесконечная группа, бесконечно размерная линейная группа, плотное семейство подгрупп, локально разрешимая группа, конечная центральная размерность.*