CURRENT ISSUES IN ENSURING THE QUALITY OF MATHEMATICAL EDUCATION

MONOGRAPH



BUDAPEST

Tarasenkova, N. (Eds). (2019). ***Current Issues in Ensuring the Quality of Mathematical Education:*** *Monograph.* In L. Kyba (A. Ed.). Budapest, Hungary: SCASPEE.

The Authors:
I. Akulenko, D. Bobyliev, T. Bodnenko, M. Burda, S. Cernajeva, O. Chernobai, Z. Kravchenko, V. Kirman, T. Krylova, L. Kulyk, I. Lovyanova, I. Malova, O. Plysiuk, Z. Serdiuk, I. Sitak, N. Tarasenkova, D. Tinkova, K. Vlasenko, I. Volodko, A. Yakovenko, & O. Zydkov

Scholarly Editor: Dr. habil., Full Prof. N. Tarasenkova
Author's Editor: L. Kyba

ISBN 978-615-00-7209-8
SCASPEE, Budapest, 2019

DOI: http://doi.org/10.31174/SEND-MON003

This book will be of interest to all researchers in the field of didactics of mathematics.

© Authors & Editors, 2019
© SCASPEE, 2019
All rights reserved.
Extracts may be reproduced for non-commercial purposes provided that the source is fully acknowledged.
All requests for the reproduction of this work in whole or in part must be addressed to SCASPEE, Budapest

Editorial board
Editor-in-chief: Dr. Xénia Vámos
Honorary Senior Editors:

Jenő Barkáts, Dr. habil. **Nina Tarasenkova, Dr. habil.**

Andriy Myachykov, PhD in Psychology, Senior Lecturer, Department of Psychology, Faculty of Health and Life Sciences, Northumbria University, Northumberland Building, Newcastle upon Tyne, United Kingdom

Edvard Ayvazyan, Doctor of Science in Pedagogy, National Institute of Education, Yerevan, Armenia

Ireneusz Pyrzyk, Doctor of Science in Pedagogy, Dean of the Faculty of Pedagogical Sciences, University of Humanities and Economics in Włocławek, Poland

Irina Malova, Doctor of Science in Pedagogy, Head of the Department of Methodology of Teaching Mathematics and Information Technology, Ivan Petrovsky Bryansk State University, Russia

Kosta Garow, PhD in Pedagogy, Associate professor, Plovdiv University «Paisii Hilendarski», Bulgaria

Marian Wloshinski, Doctor of Science in Pedagogy, Faculty of Pedagogical Sciences, University of Humanities and Economics in Włocławek, Poland

Oleg Melnikov, Doctor of Science in Pedagogy, Belarusian State University, Belarus

Roza Uteeva, Doctor of Science in Pedagogy, Head of the Department of Algebra and Geometry, Togliatti State University, Russia

Tatyana Prokhorova, Doctor of Science in Pedagogy, Professor of Psychology, Astrakhan State University, Russia

Vasil Milloushev, Doctor of Science in Pedagogy, Professor of the Department of Mathematics and Informatics, Plovdiv University «Paisii Hilendarski», Plovdiv, Bulgaria

Veselin Kostov Vasilev, Doctor of Psychology, Professor and Head of the Department of Psychology, Plovdiv University «Paisii Hilendarski», Bulgaria

Zoltán Poór, CSc in Language Pedagogy, Head of the Institute of Pedagogy, Apáczai Csere János Faculty of the University of West Hungary, Hungary

THE JOURNAL IS INDEXED IN:

INDEX COPERNICUS: ICV 2014: 70.95; ICV 2015: 80.87; ICV 2016: 73.35; **ICV 2018: 90.25**
SCIENTIFIC JOURNAL IMPACT FACTOR: 2013: 2.642; 2014: 4.685; 2015: 5.278; 2016: 6.278; 2017: 6.391; 2018: 6.538
GLOBAL IMPACT FACTOR (GIF): 2013: 0.545; 2014: 0.676; 2015: 0.787
GOOGLE SCHOLAR: citations 1456; h-index 9
ULRICHS WEB GLOBAL SERIALS DIRECTORY
UNION OF INTERNATIONAL ASSOCIATIONS YEARBOOK
SCRIBD
ACADEMIA.EDU

Cite this book:

APA Tarasenkova, N. (Eds). (2019). *Current Issues in Ensuring the Quality of Mathematical Education: Monograph.* In L. Kyba (A. Ed.). Budapest, Hungary: SCASPEE.

SSU Current Issues in Ensuring the Quality of Mathematical Education: monograph / I. Akulenko, D. Bobyliev, T. Bodnenko, M. Burda, S. Cernajeva, O. Chernobai, Z. Kravchenko, V. Kirman, T. Krylova, L. Kulyk, I. Lovyanova, I. Malova, O. Plysiuk, Z. Serdiuk, I. Sitak, N. Tarasenkova, D. Tinkova, K. Vlasenko, I. Volodko, A. Yakovenko, & O. Zydkov; Eds. prof. N. Tarasenkova, & L. Kyba. – Budapest: SCASPEE, 2019. – 224 p.

CONTENTS

PREFACE 7

Chapter 1 Learning Mathematics at Secondary & Vocational Schools 11

1.1 Conceptual Bases for the Selection of Textbooks on Mathematics for Lyceums 12
Burda M., Tarasenkova N.

1.2 Competence Approach to Mathematics Teaching and Solving Physical Problems Graphically 25
Kulyk L., Serdiuk Z., Bodnenko T.

1.3 Development of Distance Concepts through Metric Spaces as the Implementation of Inter-subject Relations (7th – 9th high school grades) 39
Lovianova I., Bobyliev D.

1.4 Pace-of-learning-adjusted High School Mathematics: Didactic Principles of Differentiated Instruction 51
Plysiuk O.

1.5 Development of Mathematical Concepts in High School: Specifics 67
Kravchenko Z.

1.6 Modeling Elements and the Development of the Engineering Students' Mathematical Competence 79
Tinkova D.

1.7 A review of the Experience with the Implementation of STEM-education Technologies 97
Vlasenko K., Sitak I., Chumak O., Kondratyeva O.

Chapter 2 Mathematical Training at the University 111

2.1 Testing as a Method for Assessing Students' Knowledge 112
Volodko I., Cernajeva S.

2.2 Monitoring the Ukrainian University Students' Math Levels 126
Krylova T.

2.3 On the use of Algorithms in Teaching Probability Theory 138
Chernobai O.

Chapter 3 Math Teacher Training in Graduate and Postgraduate Education 153

3.1 The Enhancement of Mathematics Teacher Training by Use of Methodological Projects 154
Malova I.

3.2 Project-Based Learning and Teaching Mathematics: Theoretical Framework and Teachers' Beliefs 169
Akulenko I., Zydkov O., Yakovenko A.

3.3 Flexible Model of Training Mathematics Teachers in the System of Postgraduate Pedagogical Education 189
Kirman V.

REFERENCES 206

OUR AUTHORS 222

2.3 On the Use of Algorithms in Teaching Probability Theory*

O. Chernobai

*The article is published in the author's translation

Introduction

The theory of probabilities is a branch of mathematics that studies the patterns of random phenomena: random events, random variables, their functions, properties and operations on them. Mathematical models in the theory of probabilities describe with certain accuracy level the tests, results of which are complexly determined by testing circumstances (see e.g. Prokhorov[1], 1999).

The study of the theory of probabilities in modern conditions becomes especially relevant. This is caused by the fact that growing number of specialties require application of mathematical knowledge, practical skills and expertise of a sufficiently high level. The development of Ukrainian national high school includes the improvement of both mathematical education in general and its separate sections. The main areas of improvement are updating the content and technology of teaching higher mathematics and mathematical disciplines. In this article there will be reviewed specialties of algorithmic approach usage within teaching of the theory of probabilities, probabilistic process and mathematical statistics.

Analysis of latest publications related to the topic

In high school, depending on the specialization, the course of probability theory is either studied as an independent discipline, or is included in the course of higher and applied mathematics. The purpose, main tasks

[1] Prokhorov, Y. (1999). *Veroyatnost i matematicheskaia statistika: Entsyklopediia [Probability and mathematical statistics: Encyclopedia]*. Moscow, Russia: Bolshaia rossiiskaia entsyklopediia. [In Rus.].

and the motivation of the course were considered earlier (see e.g. abstracts papers[2] [3]).

In our report[4] it is stated that the process of learning the branch of probability theory by students has a number of features. Firstly, the scope of knowledge to be studied related to probability sections is quite large and time prescribed for these sections of higher and applied mathematics is limited. Secondly, there are a number of difficulties faced by students while studying the theory of probabilities. They are related to such aspects as presence of abstractively-logical analysis, probabilistic (ambiguous) assertions in the discipline, which are required to transform the content of the task into the language of probabilistic models for its further solution. The main difficulty is that events are less clear than figures, numbers, or expressions, and the probability and possibility are not as intuitive as length, area or volume. An event and its consequences are special types of mental objects, the mathematical formation of which is much more difficult than the formation of a figure (in geometry) or quantity (in arithmetic or algebra).

In addition, every year there is a decrease in level of mathematical preparedness of candidates for admittance to higher educational institutions. Modern students often come to an institution being unable to think logically and perform analysis. The traditional difficulty of mathematical disciplines – analysis of the tasks' text and, as a result,

[2] Chernobai, O. (2015). Pro deiaki osoblyvosti vykladannia kursu «Vyshcha ta prykladna matematyka» [About some specialties of teaching study course «Higher and Applied Mathematics»]. *Proceedings from Mathematics in modern technical university r14: III Mizhnarodna naukovo-praktychna konferentsiia (25-26 hrudnia 2014 roku) – 3rd International Scientific and Practical Conference.* (pp. 211–213). Kyiv, Ukraine: NTUU «KPI». [In Ukr.].

[3] Chernobai, O. (2017). Motyvatsiia pry vykladanni kursu vyshcha ta prykladna matematyka [Motivation within teaching of study course «Higher and Applied Mathematics»]. *Proceedings from Mathematics in modern technical university r16: Piata Mizhnarodna naukovo-praktychna konferentsiia (29-30 hrudnia 2016 roku) – 5th International Scientific and Practical Conference.* (pp. 183–184). Kyiv, Ukraine: NTUU «KPI». [In Ukr.].

[4] Chernobai, O. (2019). Alhorytmizatsiia v protsesi navchannia teorii ymovirnostei [Algorithmization in the process of teaching the theory of probabilities]. *Proceedings from Mathematics in modern technical university r18: Soma Mizhnarodna naukovo-praktychna konferentsiia (28-29 hrudnia 2018 roku) – 7th International Scientific and Practical Conference.* (pp. 197–200). Kyiv, Ukraine: NTUU «KPI». [In Ukr.].

the ability to solve the tasks in text format - is crucial in this subject as all tasks are in text format. Text tasks on the theory of probability, combinatorics, statistics, and probabilistic processes are much more diverse than algebraic ones. Apart from classical tasks (throwing up the dice, coins, randomly pulling the colored balls) there is a large number of similar contexts.

When solving a new task, to understand that this is the main task usually turns out to be quite difficult for the student. Not enough prepared students do not see an analogy, even in the tasks of pulling colored pencils or multicolored balloons out of the box.

In this regard, the teacher faces a rather difficult task of adapting students to studying his subject. One way to overcome these difficulties is to apply algorithmic approach to solving probabilistic tasks. Algorithms can be provided to students in the form of tables, sequence of actions and schemes.

The aim of the study

To develop algorithmic components of the methodical system of teaching probability theory's certain topics.

Presentation of main material

In the abstracts of papers mentioned above, it is said that one of the first topics in the course "Probability theory, probabilistic processes and mathematical statistics" is the classical definition of the probability of an event. Usually, after reading the task, the student faces chaos in his mind, everything is mixed: events, results, probabilities. The following algorithms help to structure analysis and to build a logical chain of consideration:

1 Define which experiment occurs in task being analyzed.
2 How many probable outcomes does the experiment have (n). It is good at this stage when students formulate questions aloud.
3 Introduce event A, the probability of which should be found in the task.

4 Determine how many outcomes contribute to the event (m). It is also important to formulate questions aloud here.

5 Apply the formula of classical probability $P(A) = \frac{m}{n}$.

At each stage, it is important to offer the student to formulate the question which should be answered at this stage. In doing so, one should obtain from the student a clear understanding of what is a test (experiment), what is an event, and what is the probability of the event.

Example. The group consists of 25 students, 5 of them are participants in the skills competition on Higher mathematics. What is the probability that a randomly selected student will be a participant of skills competition.

To solve the task, let's apply the algorithm.

1 Experiment: a student from the group is chosen.
2 Probable outcomes of the experiment ($n = 25$).
3 Event A – «randomly selected student is a participant of skills competition on higher mathematics».
4 Outcomes contributing to the event A ($m = 5$).
5 According to the known formula of the classical probability definition, we obtain a numerical value $P(A) = \frac{m}{n} = \frac{5}{25} = \frac{1}{5}$.

The classical definition of probability cannot be applied to an experiment with an infinite number of equally possible consequences. In this case, the geometric determination of probability is used. Within analysis a number of features, such as the elementary event in the experiment can be limited to the choice of the point, elementary events are equally possible, the number of elementary events is infinite, and their number forms a finite dimensional area, direct the student to the conclusion on possibility of applying a geometric probability – the probability of point falling within an area (section, part of the plane, etc.). The solution of the task, in comparison with the previous ones, is connected with the

necessity of interpreting the experiment as a point choice in a certain range.

The algorithm for solving tasks on the geometric probability can be formulated as follows:

1 Define which experiment occurs in task being analyzed. As the number of outcomes of experiment defined in the task is infinite, geometric approach should be applied to probability calculation. For this purpose, experiment should be defined as point choosing in certain range.
2 Define G range of all possible outcomes and find its measure (length, area or volume) – $mes\ G$.
3 Introduce event A, the probability of which should be found in the task. Define the area Q, which is a subset of the set G, and is the set of results contributing to the event A. Find the measure of the Q set – $mes\ Q$.
4 Find the probability of the event A by the formula:

$$P(A) = \frac{mes\ Q}{mes\ G}.$$

Solving tasks on geometric probability causes a lot of difficulties. This is due to the difficulty of interpreting the task in text format in a way of putting a point into a certain area. In this case, it is inappropriate for a teacher to immediately give students the idea of this interpretation, but instead with the help of a series of questions he should stimulate the appearance of right idea within the students by themselves.

Example. Two students agreed to meet in a certain place in the time interval from t_1 to t_2 hours, as well as that the one who comes first will wait for the second within t_3 hours. Find the probability that the meeting will occur if each person can arrive at any time $t_3 \in [t_1; t_2]$.

Let's apply the algorithm to solve the task.

1 Experiment: two persons are located in a certain place within a certain time frame.
2 G range of all possible outcomes – set of points of a square with a side $t_2 - t_1 = T$.
3 Event A – meeting will occur.

4 The area Q corresponds to the shaded part of the square, if the moment of arrival of each person – A will take place under conditions $|x-y| \le t_3$, where $0 \le x \le T, 0 \le y \le T$.

5 These conditions are represented in the XOY coordinate system. The set of all results corresponds to the area of the square $ONCK$, and A event – area of the hexagon $OEDCBA$.

Using the geometric definition of probability, we will get:

$$P(A) = \frac{S_{OEDCBA}}{S_{ONCK}} = \frac{T^2 - (T - t_3)^2}{T^2} = \frac{T^2 - T^2 + 2Tt_3 - t_3^2}{T^2} = \frac{t_3(2T - t_3)}{T^2}.$$

The mentioned task was considered in the worksheet on probability theory of M. Semko[5] (2011).

Within study of axioms and theorems on probability theory, it is advisable to present them in the form of the following tables (see e.g. Chernobai[6], 2018):

To solve tasks using the theorems of addition and multiplication of probabilities, we propose the following algorithm:

1 Formulate an event, the probability of which must be found in the task.
2 Formulate an event through which you can express the sought event by adding, multiplying, and subtracting events.
3 Find the probability of an event formulated in p. 2.
4 Express the sought event through the event formulated in p. 2, by adding, multiplying and subtracting events.
5 Pass on to the probability of the sought event and apply the addition and multiplication probability theorems.

[5] Semko, M., Zadorozhnia, T., Kuchmenko, S., Mamonova, H., Rudenko,I., & Chernobai, O. et.al. (2011). *Teoriia ymovirnostei ta matematychna statystyka (robochyi zoshyt dlia indyvidualnykh zaniat) [Probability theory and mathematical Statistics (workbook for individual tasks)]*. Kyiv, Ukraine: Instytut matematyky NAN Ukrayiny. [in Ukr.].

[6] Chernobai, O. (2018). Osoblyvosti vykladannia teorii ymovirnostei u suchasnykh umovakh. [Specialities of probability theory teaching in modern conditions]. *Proceedings from Accounting and taxation: realities and perspectives r18: III Vseukrainska naukovo-praktychna internet-konferentsiia (18-20 kvitnia 2018 roku) – 3rd All-Ukrainian Scientific and Practical Internet-conference.* (pp. 665–666). Irpin, Ukraine: UDFSU. [in Ukr.].

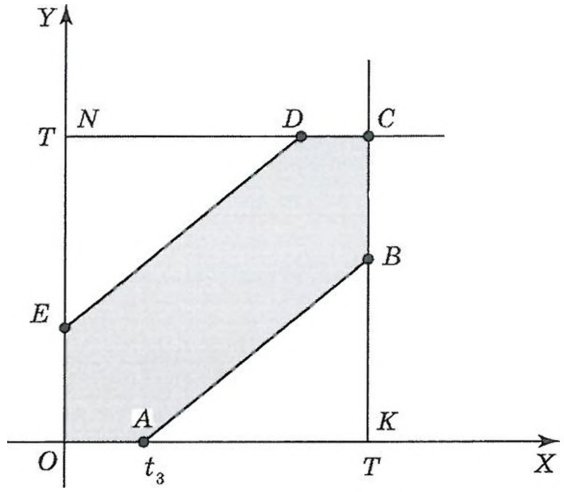


Figure 2.3.1 Geometrical presentation of task conditions.

Example. To complete the task, the manager addresses two independent performers. The probability that the first performer will execute the task is equal to 0.7 and the second – 0.8. Find probability that the task will be executed.

Let's solve the task using the above-mentioned algorithm:

1 Let's define event A – «the manager's task is completed».
2 Let event A_1 be «task is completed by first performer».
3 Event A_2 «task is completed by second performer».
4 We find the probability of the sought event, using the sum of the probabilities of compatible but independent events $P(A) = P(A_1) + P(A_2) - P(A_1 A_2)$ (refer to Table 2.3.1).
5 Thus, the probability of event A is equal to:

$$P(A) = P(A_1) + P(A_2) - P(A_1 A_2) = 0{,}7 + 0{,}8 - 0{,}56 = 1{,}5 - 0{,}56 = 0{,}94.$$

Example. Three students take the exam in higher mathematics. The probability to pass the exam with excellent

$P(A+B)$	
A and B are mutually exclusive	A and B are compatible
Probability that only one event will occur	Probability that at least one event will occur
$P(A+B) = P(A) + P(B)$	$P(A+B) = P(A) + P(B) - P(AB)$

Table 2.3.1 Probability of the sum of two events.

$P(A_1 + A_2 + \ldots A_n)$	
Mutually exclusive events	Compatible events
Probability that only one event will occur	Probability that at least one event will occur
$P(A_1 + A_2 + \ldots A_n) =$ $= P(A_1) + P(A_2) + \ldots P(A_n)$	$P(A_1 + A_2 + \ldots + A_n) =$ $= 1 - P(\overline{A_1} \cdot \overline{A_2} \cdot \ldots \overline{A_n})$ $A = A_1 + A_2 + \ldots + A_n$ $\overline{A} = \overline{A_1} \cdot \overline{A_2} \cdot \ldots \overline{A_n}$

Table 2.3.2 Probability of the sum of several events.

mark for the first student is 0.5, for the second one — 0.3, and for the third one — 0.2. Find the probability that three students will pass the exam with excellent mark.

Let's apply the algorithm for solving tasks based on the theorem of independent events' probability multiplication (refer to Table 2.3.3).

1 Let's formulate event A — «all three students will pass the exam with excellent mark».

2 Let event A_1 —« first student will pass the exam with excellent mark»,
event A_2 —« second student will pass the exam with excellent mark»,
event A_3 —« third student will pass the exam with excellent mark». Then sought event can be presented as product $A = A_1 \cdot A_2 \cdot A_3$.

3 The probability of event A can be found by the formula for multiplying the probabilities of independent events:

$$P(A) = P(A_1) \cdot P(A_2) \cdot P(A_3).$$

4 So the probability of event:

$$P(A) = P(A_1) \cdot P(A_2) \cdot P(A_3) = 0{,}5 \cdot 0{,}3 \cdot 0{,}2 = 0{,}03.$$

Some principles and examples of using the algorithms of the course have been earlier considered by the author. And now we suggest an algorithm for solving a task using formula of total probability and Bayes formula. The formula of total probability is a consequence of the addition and multiplication theorems. For these formulas, students can be offered the following algorithm:

$P(A_1 \cdot A_2 \cdot \ldots A_n)$	
Probability of occurrence of all events together	
Independent events	Dependent events
$P(A_1 \cdot A_2 \cdot \ldots A_n) =$ $= P(A_1) \cdot P(A_2) \cdot \ldots P(A_n)$	$P(A_1 \cdot A_2 \cdot \ldots \cdot A_n) =$ $P(A_1) \cdot P_{A_1}(A_2) \cdot P_{A_1A_2}(A_3) \cdot$ $\cdot \ldots \cdot P_{A_1A_2 \ldots A_{n-1}}(A_n)$

Table 2.3.3 Probability of the product of events.

1 Formulate an event A, the probability of which must be found in the task (or for Bayes formula – event, which occurred as a result of experiment).

2 Formulate hypothesis $H_1, H_2, \ldots H_n$.

3 Find probability of hypothesis $P(H_1), P(H_2), \ldots P(H_n)$.

4 Perform check using formula $P(H_1) + P(H_2) + \ldots P(H_n) = 1$.

5 Define total probability formula for defined task:

$$P(A) = \sum_{i=1}^{n} P(H_i) \cdot P_{H_i}(A).$$

6 Find conditional probabilities $P_{H_i}(A)$. At this stage it is preferable that students at least orally define the probability of which event and under which conditions they are searching for.

7 Substitute the values found in points 3 and 6 in the formula of total probability (point 5).

For the Beyies formula, one more point is added (point 8).

8 Calculate the probability of the sought-after hypothesis with the Bayes formula:

$$P(H_i) = \frac{P(H_i) P_{H_i}(A)}{P(A)}.$$

Example. The department of the SFS of Ukraine receives information from three independently operating centers. From the first center comes 50% of information, from the second — 30%, and from the third — 20%. The probability of error while processing information from the first center is equal to 0,1; from the second — 0,05, and from the third — 0,15. What is the probability of receiving information with

an error? Information was received with error, calculate the probability that it was received from the third center.

Let's solve the task using the above-mentioned algorithm:

1 Formulate an event A – «information was received with error».

2 Formulate relevant hypothesis:

H_1 – information was received from the first center,

H_2 – information was received from the second center,

H_3 – information was received from the third center.

3 Find probability of hypothesis:

$$P(H_1) = 0{,}5,\ P(H_2) = 0{,}3,\ P(H_3) = 0{,}2.$$

4 Perform check using formula:

$$P(H_1) + P(H_2) + P(H_3) = 0{,}5 + 0{,}3 + 0{,}2 = 1.$$

5 Define total probability formula for defined task:

$$P(A) = \sum_{i=1}^{3} P(H_i) \cdot P_{H_i}(A).$$

Find conditional probabilities:

$$P_{H_1}(A) = 0{,}1;\ P_{H_2}(A) = 0{,}05;\ P_{H_3} = 0{,}15.$$

6 Substitute the values found in the formula of total probability. We have:

$$P(A) = \sum_{i=1}^{3} P(H_i) \cdot P_{H_i}(A) = 0{,}5 \cdot 0{,}1 + 0{,}3 \cdot 0{,}05 + 0{,}2 \cdot 0{,}15 = 0{,}095.$$

7 Calculate the probability of the sought-after hypothesis with the Bayes formula:

$$P(H_i) = \frac{P(H_i)P_{H_i}(A)}{P(A)} = \frac{0{,}2 \cdot 0{,}15}{0{,}095} = 0{,}316.$$

Consequently, the probability that information with error was received from the third center is 0.316.

For tasks in which there is a series of tests under the Bernoulli scheme, the following algorithm can be set:

1 Formulate an event A, the probability of which must be found in the task.

2 Set Bernoulli scheme:

- define what should be treated as one test,
- define quantity of tests n,
- check if tests are independent,
- split solutions of one tests into two groups: «success» and «fail». «Success» = {solutions, which contribute to event A}, «fail» = {solutions opposite to «success»},
- find probability of «success» – p i «fail» – q. It should be checked that p and q do not change from test to test in this series of tests.

3 To express the probability of event A via the probability of m success in n tests performed under the Bernoulli scheme $P(A) = P_n(m)$.

4 Apply the Bernoulli formula to p. 3 $P_n(m) = C_n^m p^m q^{n-m}$ or in case quantity of tests is big, the approximate formulas: if n is big and p is very small ($np < 10$) – Poisson formula:

$$P_n(m) \approx \frac{\lambda^m e^{-\lambda}}{m!}, \quad \lambda = np,$$

if n is big and p is not very small ($np \geq 10$) – Moivre–Laplace formula:

$$P(m) \approx \frac{1}{\sqrt{npq}}\varphi(x), \quad x = \frac{m - np}{\sqrt{npq}}.$$

Example. Out of 25 people, 8 are entitled to a tax benefit. What is the probability that two out of three randomly selected individuals will be entitled to tax benefits?

Let's solve the task using the corresponding algorithm.

1 Formulate an event A – «two out of three randomly selected individuals are entitled to tax benefits».

2 Set Bernoulli scheme:

- one test is to select one person out of 25,
- define quantity of tests $n = 3$,
- tests are independent,

- 8 cases contribute to «Success» of test, 17 correspond to «fail»,
- probability of «success» $p = \frac{8}{25} = 0{,}32$; and probability of «fail» is $q = 1 - p = 1 - 0{,}32 = 0{,}68$.

3 Express the probability of event A as per Bernoulli formula: $P(A) = P_3(2)$.

4 Calculate the probability using the Bernoulli formula:

$$P_3(2) = C_3^2 p^2 q^1 = \frac{3!}{2!(3-2)!}(0{,}32)^2(0{,}68)^1 = 0{,}208.$$

The task was considered in the article of T. Zadorozhnia[7] (2016).

Let's formulate a task which can be solved using the approximate formulas (see e.g. study guides of O. Bashchuk[8], 2019; I. Rudenko[9], 2017).

Example. The firm, which performs repair of apartments, puts the promo leaflets in mailboxes. Previous experience has shown that about in nine out of a thousand cases orders will be placed for repair of apartments. Find the probability that when 300 leaflets are placed, the number of orders will be equal to two.

[7] Zadorozhnia, T., Kharenko, S., Kuchmenko, S., Chernobai, O., Bashchuk, O., & Skaskiv, L. et al. (2016). Zadachi pro podatky [Tasks about taxation]. *Matematyka v ridnii shkoli. – Mathematics in home school, 10.* 16-21. [In Ukr.].

[8] Bashchuk, O., Kuchmenko, S., Skaskiv, L., & Chernobai, O. (2019). *Vyshcha ta prykladna matematyka: zbirnyk vprav ta zadach [Higher and applied mathematics: collection of exercises and tasks].* Irpin, Ukraine: Universytet DFS Ukrainy. [In Ukr.].

[9] Rudenko, I., & Chernobai, O. (2017). *Vyshcha ta prykladna matematyka [Higher and applied mathematics].* Irpin, Ukraine: Universytet DFS Ukrainy. [In Ukr.].

Let's apply the algorithm of Bernoulli scheme.

1 Formulate an event A – «when 300 leaflets are placed, the number of orders is equal to two».

2 Set Bernoulli scheme:

- one test is to place promo leaflets,
- define quantity of tests $n = 300$,
- tests are independent,
- 9 cases out of 1000 contribute to «Success» of test,
- probability of «success» $p = 0{,}009$; and probability of «fail» is $q = 1 - p = 1 - 0{,}009 = 0{,}991$.

3 Express the probability of event A as per Bernoulli formula $P(A) = P_{300}(2)$.

4 In our case $n = 300$ is quit big and $p = 0{,}009$; is very small and $np = 2{,}7 < 10$, therefore Poisson formula is applied:

$$P_n(m) \approx \frac{\lambda^m e^{-\lambda}}{m!}, \quad \lambda = np = 2{,}7,$$

$$P_{300}(2) \approx \frac{2{,}7 \cdot e^{-2{,}7}}{2!} \approx \frac{2{,}7 \cdot 0{,}067}{2} \approx 0{,}09045.$$

Let's review the task, being solved applying Moivre–Laplace theorem.

Example. According to statistics, 2% of residents of certain city who rent out apartments do not pay taxes. Find the probability that 300 residents do not pay taxes out of 5000 residents who rent out apartments.
Let's solve the task applying boundary theorems of Bernoulli scheme.

1 Formulate an event A – «300 residents do not pay taxes out of 5000 residents who rent out apartments».

2 Set Bernoulli scheme:

- one test is renting out apartments,
- define quantity of tests $n = 5000$,
- tests are independent,
- 2 cases out of 100 contribute to «Success» of test,
- probability of «success» $p = 0,02$; and probability of «fail» is $q = 1 - p = 1 - 0,02 = 0,98$.

3 Express the probability of event A as per Bernoulli formula $P(A) = P_{5000}(300)$.

In our case $n = 5000$ is quit big, and probability $p = 0,02$ and $np = 100 \geq 10$, therefore Moivre–Laplace formula is applied.

Firstly, we find:

$$x = \frac{m - np}{\sqrt{npq}} = \frac{300 - 10}{\sqrt{5000 \cdot 0,02 \cdot 0,98}} = \frac{290}{98} = 2,9591.$$

Corresponding probability:

$$P_{5000}(300) \approx \frac{1}{\sqrt{npq}}\varphi(x) = \frac{1}{98}\varphi(2,9591) = \frac{0,051}{98} = 0,0005.$$

Conclusions

Algorithmization of the task-solving process on probability theory helps students to clearly see a plan for solving the task, analyze the task conditions, teaches them to think analytically, logically and in a structured way. This approach contributes to a better acquisition of knowledge, a more clear and conscious application of the basic concepts and theorems of probability theory. Besides, the use of algorithms does not only enable to teach students to solve tasks, but also develops the ability to analyze tasks and solve them not only by example, to draw concrete conclusions and to summarize the results.

Considered algorithms with the use of classical probability definition, theorems of addition and multiplication of probabilities, Bernoulli scheme can be also applied by mathematics teachers of general education institutions while teaching the theory of probabilities.