

The Negative Derivative Feedback Controller Tuning for Vibration Suppression

Volodymyr PUZYROV ^{a,b}, Nataliya LOSYEVA ^{a,b}, Nina SAVCHENKO ^{c,1} and Daria TERMENZHY ^d

^a *Universidad de Barcelona, Spain*

^b *Nizhyn Mykola Gogol State University, Ukraine*

^c *National Aerospace University, Ukraine*

^d *State Tax University, Ukraine*

Abstract. This paper investigates the problem of suppressing vibrations induced by external harmonic excitation using a Negative Derivative Feedback Controller (NDFC). The relevance of this topic is driven by the growing demand for improved vibration control in lightweight and highly flexible engineering structures, which are increasingly used in aerospace, mechanical, and civil applications. Such structures are particularly sensitive to resonance phenomena due to their low inherent damping, leading to performance degradation and potential failure. A simplified single-degree-of-freedom (DoF) model of the main structure with an attached NDFC is considered. Two independent analytical procedures for tuning the controller are proposed, aimed at ensuring effective vibration mitigation. These procedures allow for a straightforward identification of optimal regions in the parameter space, particularly with respect to damping and cut-off frequency. The use of the NDFC provides an efficient means for passive vibration suppression. Numerical simulations confirm the effectiveness of the proposed methods and demonstrate good agreement with analytical predictions. The results contribute to the advancement of vibration control strategies, offering a practical and computationally accessible approach for enhancing the dynamic performance of flexible structures.

Keywords. Negative derivative feedback controller, stability, frequency-amplitude dependence, peak responses

1. Introduction

Industrial developments in recent decades have increased the requirements for vibration control, especially for lightweight and highly flexible structures [1,2]. The main problem with such systems is the rise of oscillations amplitude in the vicinity of resonant frequencies, mainly due to the low damping coefficient associated with the first structural modes. It may lead to a degradation of system performance and cause disruptions and breakdowns. One of the fruitful ideas to prevent such hazard was proposed by Fanson and Caughey [3] which suggests that the displacement is fed back with a positive sign

¹ Nina SAVCHENKO, Corresponding author, National Aerospace University, Ukraine; E-mail: n.savchenko@khai.edu.

through a second-order dynamic system. With the similar motivation in recent years the negative position feedback (NPF) [4] was suggested. This approach was tested experimentally [5] and shown its effectiveness. Jamshidi and Collete [6] proposed the design of NDF filter based on H_2 and H_∞ method for collocated systems. A positive position feedback (PPF) and negative derivative feedback (NDF) controllers are applied for reducing the vibrations of a cantilever beam [7]. Comparative analysis of PPF and NDF schemes as well as an experimental setup were presented in [8]. The theoretical methodology by using the averaging method for getting a perturbed solution was applied in [9].

In this paper, two independent procedures for tuning the NDF controller following the analytical scheme are discussed. The approach is quite simple and effective.

2. Formulation of the Problem

The system under study consists of a main structure modeled by a mass m with stiffness k and an attached NDF controller. An external force F_e is applied on the mass, and the NDF controller is producing an action force $F_a = k_c u$ which is dependent on the system displacement (figure 1). This system can be described by equations

$$m \frac{d^2 \tilde{x}}{dt^2} + k \tilde{x} = A_e \cos \omega t + F_a, \quad \frac{d^2 u}{dt^2} + \xi_c \omega_c \frac{du}{dt} + \omega_c^2 u = -\omega_c \frac{d \tilde{x}}{dt}. \quad (1)$$

where $\tilde{x}$ is the mass displacement, u is the controller signal, ξ_c , ω_c and k_c are the damping ratio, cut-off frequency and gain of NDF controller respectively.

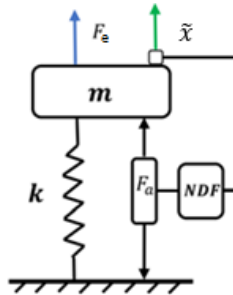


Figure 1. Schematic view of mechanical system.

Let us introduce the dimensionless variables and parameters according to formulas

$$\tilde{x} = \frac{A_e}{m} x, \quad \alpha = \omega_c / \omega_0, \quad \beta = \frac{k_c}{k \xi_c}, \quad \eta = \xi_c^2, \quad \omega_0 = \sqrt{\frac{k}{m}}, \quad \tau = \omega_0 t, \quad \Omega = \frac{\omega}{\omega_0}. \quad (2)$$

The amplitude of steady state harmonic responses is $F(\Omega^2) = P/Q$, where

$$P(q, \alpha, \eta) = q^2 + \alpha q (\eta - 2) + \alpha^2$$

$$Q(q, \alpha, \beta, \eta) = q^4 + q^3 (\alpha \eta - 2\alpha - 2) + q^2 [1 + \alpha^2 + 4\alpha - 2\alpha \eta (1 + \beta)] - \alpha q [2(\alpha + 1) - \eta (1 + \beta)^2] + \alpha^2. \quad (3)$$

Note that, unlike the situation with the use of a dynamic absorber, for which the operating mode is always stable, in the case of NDF controller the instability is possible. The characteristic equation of the system (1) has the form

$$\lambda^4 + \eta \alpha \lambda^3 + (1 + \alpha^2) \lambda^2 + \eta \alpha (1 + \beta) \lambda + \alpha^2 = 0.$$

According to the criterion for asymptotic stability, it is necessary and sufficient to satisfy the conditions

$$1 + \beta > 0, \quad \beta (\alpha^2 - 1 - \beta) > 0$$

which may be separated on two subcases:

$$\alpha^2 - 1 < \beta < 0, \alpha < 1 \quad \text{or} \quad 0 < \beta < \alpha^2 - 1, \alpha > 1. \quad (4)$$

3. Analysis of the Frequency-Amplitude Relation

Assuming, that system's mass and stiffness are given, we seek the parameters of controller which allow to reduce the maximal level of possible oscillation amplitude in some range of system's frequencies. With respect to dimensionless parameters this means that we have to suggest some optimal choice for vector (α, β, η) . This goal may be achieved in different ways, and we suggest here two independent approaches.

The first one uses the classical idea of "peak equalizing" proposed by Den Hartog [10] and widely used in literature [11 – 13]. Geometrically this means that two peaks are of the same altitude, thus have the common tangent line (figure 2). Denote this altitude by H and consider the equality $\frac{P(q)}{Q(q)} = H$. The latter may be presented in the following form

$$\tilde{P}(q) \triangleq Q(q) - \frac{1}{H} P(q) = 0. \quad (5)$$

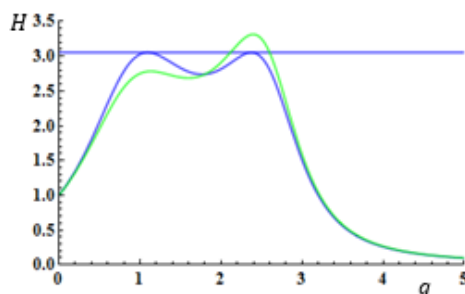


Figure 2. Peak equalizing: $\alpha = 3.17$; $\beta = 1.2$; $\eta = 0.48$ (green line), $\eta = 0.44$ (blue line).

Points of local maximums of polynomial $\tilde{P}(q)$ coincide with its zeros, which means that these zeros are of multiplicity two. Therefore, polynomial $\tilde{P}(q)$ may be presented in the form $\tilde{P}(q) = (q^2 - Mq + \alpha N)^2$, where M, N are some constants unknown at the moment. Because two forms of polynomial $\tilde{P}$ are equal identically with respect to q , using formulas (3) and equating pairs of coefficient on each power of q , we come to the following relations

$$M = \alpha + 1 - \frac{1}{2} \alpha \eta, \quad H = \frac{1}{1 - N^2}, \quad \eta = \frac{2(1 - N)(\alpha + N)}{\beta(2 + \beta) - N(N - \alpha)}, \quad \tilde{P}(\alpha, \beta, \eta, N) = 0, \quad (6)$$

Where $\tilde{P}$ is polynomial of ninth order (which is bulky enough to be presented here).

It is obvious that if we want to achieve a decrease in the value of H , we should minimize the value of N by choosing the parameters α, β appropriately, while maintaining positivity of η and without violate the stability conditions. This problem can be considered as a formal search for the smallest value of an implicitly defined function $N(\alpha, \beta)$ with additional restrictions (which leads to rather cumbersome calculations).

However, it is much simpler to use the fact that $0 \leq N < 1$ and directly check whether the equality $\check{P}=0$ has solutions for the value $N = 0$.

It leads to

$$\alpha^4 - 2\alpha^3\beta(\beta + 2) + 2\alpha^2\beta(-2\beta^2 + 5\beta + 2) + 2\alpha\beta^2(\beta^2 + 4) + \beta^2 - (\beta + 2)^2 = 0. \quad (7)$$

The corresponding curve is shown in figure 3a. One can see that it is possible to minimize the value of H to 1 only for values of α greater than ≈ 5.44 and $\beta > \approx 2$.

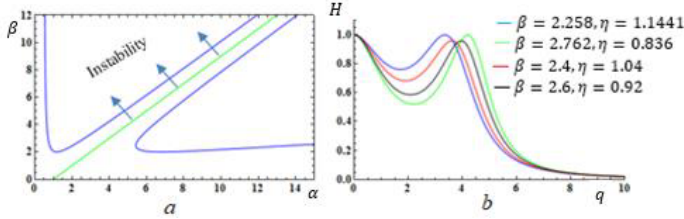


Figure 3. a: Curve (7), right branch corresponds to stable regime; b: dimensionless FA responses ($\alpha = 5.5$).

In figure 3b there are shown some curves for different pairs (α, β) satisfying (7).

Other approach is based on direct analysis of the expression P/Q . Let us determine conditions which guarantee that the magnitude of $F(q)$ will not exceed the value of 1. This can be done as follows. The condition $F < 1$ can be represented as

$$Q(q) - P(q) = qP_1(q) > 0, \quad (8)$$

where

$$P_1 = q^3 + Aq^2 + Bq + C, \quad A = -2(\alpha + 1) + \alpha\eta, \\ B = \alpha[\alpha + 4 - 2\eta(\beta + 1)], \quad C = \alpha[\beta\eta(\beta + 2) - 2\alpha]. \quad (9)$$

Our goal, therefore, is to find the conditions under which $P_1(q)$ has no positive roots. Let's analyze sequentially possible cases in the parameter space α, β, η .

It is obvious that if $C < 0$, then $P_1(q)$ has a positive root (Bolzano Theorem), therefore we will require the condition $C \geq 0$, that is

$$\eta \leq \eta_c = \frac{2\alpha}{\beta(2 + \beta)}.$$

The latter is possible only in the case of $\alpha > 1$, since otherwise, taking into account the stability condition, we have $\beta < 0, 2 + \beta > 0$, hence $\eta_c < 0$ (but η should be positive).

1.1) $C = 0, B < 0$ – there is at least one positive root ($\lim_{q \rightarrow +0} P_1(q) = -0$);

1.2) $C = 0, B \geq 0, A < 0, B \leq \frac{1}{4}A^2$ – there is at least one positive root because $P_1(\frac{1}{3}(-A + \sqrt{A^2 - 4B})) \leq 0$.

1.3) $C = 0, B \geq 0, A < 0, B > \frac{1}{4}A^2$ – $P_1(q)$ does not have positive roots because $q(q^2 + Aq + B)$ is strictly positive with $q > 0$. In dimensionless physical parameters, this case corresponds to a region bounded by curves

$$\frac{\alpha^2}{\alpha + 1} < \beta(2 + \beta), \quad \alpha(\beta^2 - 2\beta - 4) + 4\beta(\beta + 2) \geq 0, \\ \alpha^4 - 2\alpha^3\beta(\beta + 2) + 2\alpha^2\beta(2\beta^2 + 5\beta + 2) - 2\alpha\beta^2(\beta + 2)^2 + \beta^2(\beta + 2)^2 < 0.$$

The corresponding region is shown in figure 4a.

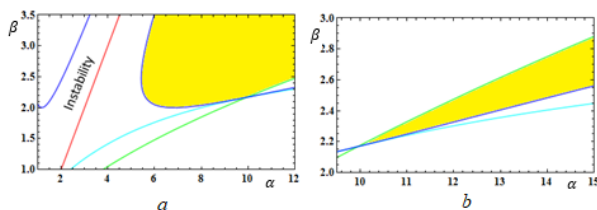


Figure 4. Domains where cases 1.3 (a) and 1.4 (b) stand.

1.4) $C = 0$, $A \geq 0$, $B \geq 0$. In this case again there are no positive roots because $P_1(q)$ is increasing function to $q > 0$. Note that this case is valid only for sufficiently large values to the parameter α (figure 4b, $\alpha > 9.948$).

2.1) $A \geq 0$, $B < 0$. Then $D_1 = A^2 - 3B > 0$, and $P_1(q)$ has two critical points $q_{1,2} = \frac{1}{3}(-A \pm \sqrt{D_1})$. q_1 is negative (local maximum), and q_2 is positive (local minimum). The function $P_1(q)$ is decreasing on interval (q_1, q_2) and increases when $q > q_2$. Thus it has absolute minimum at $q = q_2$ ($q > 0$). Obviously, the requirement $F < 1$ is fulfilled if and only if $P(q_2) > 0$ or

$$2A^3 - 9AB + 27C > D_1^{3/2}. \quad (10)$$

In the case under consideration the condition

$$2A^3 - 9AB + 27C > 0 \quad (11)$$

is fulfilled, and without loss of generality, we can rewrite this inequality in rational form, namely

$$4A^3C - A^2B^2 - 18ABC + 4B^3 + 27C^2 > 0. \quad (12)$$

The expression on the left side is the discriminant of the polynomial $P_1(q)$ taken with the opposite sign.

Thus, in this case the necessary and sufficient condition for $F < 1$ is $D < 0$. According to formulas (9) we have the following expression

$$D = \alpha \eta \{4\alpha^3 \eta^3 + 4\alpha^2 \eta^2 [(1-\beta)\alpha^2 + 2\alpha(\beta^2 - 4) - \beta^3 - \beta^2 + 10\beta + 4] + \alpha \eta [\alpha^4 + 8\alpha^3(2\beta - 3) + 2\alpha^2(-31\beta^2 + 14\beta + 44) + 8\alpha(9\beta^3 + 4\beta^2 - 20\beta - 12) - 27\beta^4 - 36\beta^3 + 76\beta^2 + 80\beta + 16] - 4(\alpha - 2)^3(\alpha - \beta)(\alpha - \beta - 2)\} \quad (13)$$

2.2) $A < 0$, $B < 0$. This case is similar to the previous one with the difference that now the expression in left-hand side of (11) can be negative, that is, formally it is necessary to analyze the system of inequalities (12), (11). However actually, the fulfillment of condition (12) entails the fulfillment of (11). This may be shown without any additional transformation, only by logical reasoning. Indeed, if expression (13) is negative (which is necessary condition for (11)), then discriminant of P_1 is negative, therefore this cubic polynomial has only one real root. As $P_1(-\infty) = -\infty$ and $P_1(0) = C > 0$, this root is negative, in other words there are no positive roots which leads to $P_1(q_2) > 0$, thus (11) is valid (which is not possible if the left-hand side is not positive). So, we see that inequality (13) is also sufficient for requirement $F < 1$.

The domain bounded by surfaces $D = 0$, $C = 0$ and subject to stability conditions is shown in figure 5. As can be seen, with the growth of the parameter α , the possibility of varying the parameters β and η significantly expands.

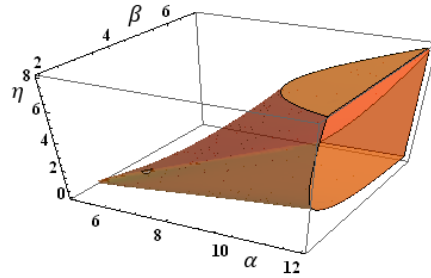


Figure 5. Domain in parameter space where condition $F < 1$ holds.

The figure 6 shows the "admissible" areas for different values of α . Also, figure 7 shows that for different sets the value of the maximum amplitude can be less than 0.5, although the value of the resonant frequency changes (and the area below the frequency-amplitude curve varies).

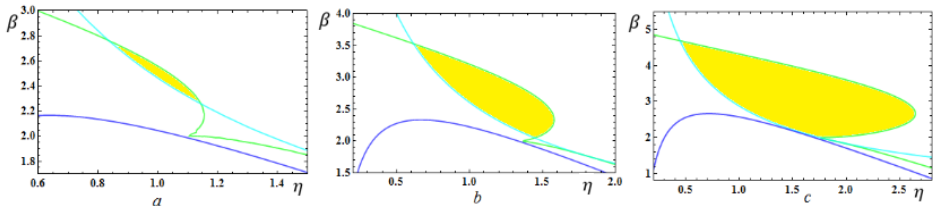


Figure 6. Fulfilment of conditions $C > 0, D < 0$ for different values of α . Respectively (a, b, c) : $\alpha = (5.5, 6, 7)$.

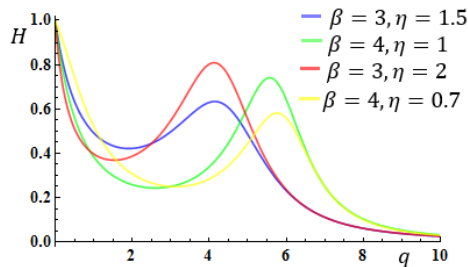


Figure 7. The influence of different parameters β, η on the FA curve shape ($\alpha = 7$).

4. Conclusion

The problem of NDFC tuning for the purpose of suppressing vibrations of a single-DoF system caused by an external periodic force is discussed. Two approaches based on analytical procedures are proposed, allowing to conveniently select effective regions in the space of mechanical parameters characterizing the controller. It is shown that cut-off frequency ω_c is a key tuning parameter. In particular, at values $\frac{\omega_c}{\omega_0} > 2.4$ it is possible to achieve a significant decrease in response peaks, improve the bandwidth, and an increase in the resonant frequency value.

The future work will be devoted to modifying presented results to nonlinear case.

Author Contributions

Conceptualization, Puzyrov V; Data curation, Puzyrov V and Losyeva N; Formal analysis, Puzyrov V and Termenzhy D; Methodology, Puzyrov V; Software, Puzyrov V and Savchenko N; Supervision, Puzyrov V and Losyeva N; Validation, Puzyrov V, Savchenko N and Termenzhy D; Visualization, Puzyrov V and Savchenko N; Writing – original draft, Puzyrov V; Writing – review & editing, Puzyrov V, Losyeva N and Savchenko N. All authors have read and agreed to the published version of the manuscript.

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